

# Natural numbers and two each methods about Fermat's Last Theorem proof

[Jae Yul Lee and You Jin Lee]

## Abstract

The natural numbers  $(X, Y, Z)$  are known as the Pythagorean triples in the equation  $X^2 + Y^2 = Z^2$ . When the number  $(n)$  is greater or equal 3 in the equation  $X^n + Y^n = Z^n$ , the equation cannot have non zero integer solutions. The truth is known as the Fermat's Last Theorem. The Fermat had written that he had found out the proof, but nobody could see his proof. We have considered the natural numbers  $(A = Z - Y)$  and  $(B = Z - X)$  in the equation  $X^n + Y^n = Z^n$  and we have found out two each methods about Fermat's Last Theorem proof.

## Key Words and Phrases

**MSC : 11-A99 Number Theory**

$$X^n + Y^n = Z^n$$

$$A = Z - Y, B = Z - X$$

$$X = G(AB)^{1/n} + A, Y = G(AB)^{1/n} + B, Z = G(AB)^{1/n} + A + B, X + Y - Z = G(AB)^{1/n}$$

$$\{G(AB)^{1/n} + A\}^n + \{G(AB)^{1/n} + B\}^n = \{G(AB)^{1/n} + A + B\}^n$$

When  $n=1$ ,  $G=0$  and when  $n=2$ ,  $G=2^{1/2} > 0$  and when  $n=3$ ,  $G = \text{Function}(A, B) > 0$ .

$$X = (2AB)^{1/2} + A, Y = (2AB)^{1/2} + B, Z = (2AB)^{1/2} + A + B$$

$$2AB = k^2 (k \text{ is } 1, 2, 3, \dots)$$

$$XY = k(k+A)(k+2A)/2A = k(k+B)(k+2B)/2B$$

All Pythagorean triples cannot be the power numbers.

$$(X^{n/2})^2 + (Y^{n/2})^2 = (Z^{n/2})^2$$

$$a = Z^{n/2} - Y^{n/2}, b = Z^{n/2} - X^{n/2}$$

$$(XY)^n = ab \{2a^2 + 2b^2 + 13ab + 6(a+b)(2ab)^{1/2}\}$$

$$\text{When } A=B, G = \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n}$$

We make a numerical formula with the figure  $\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\}$ .

$$\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

$$q = 2G(AB)^{1/n} / \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

$$G(AB)^{1/n} = q \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

When  $A=B$ ,  $q=1$ .

$$q = 2G(AB)^{1/n} / \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} (2^{1/n}A)$$

When the numbers  $(A,B)$  are the natural numbers,  
the figure  $\{G(AB)^{1/n}\}$  cannot be the natural number.

## Sentence

### 1. Preface

What the equation  $X^n + Y^n = Z^n$  cannot have non zero integer solutions and what the equation  $X^n + Y^n = Z^n$  cannot have the natural number solutions are equivalent in meaning.

When  $n=0$ , the equation is  $X^0 + Y^0 = Z^0$ . This equation means nothing.

When  $n=1$ , the equation is  $X+Y=Z$ . Every natural numbers  $(X,Y)$  make some natural number  $(Z)$ .

When  $n=2$ , the equation is  $X^2 + Y^2 = Z^2$ . The natural numbers  $(X,Y,Z)$  are the Pythagorean triples in the equation  $X^2 + Y^2 = Z^2$ .

When the number  $(n)$  is greater or equal 3, the equation  $X^n + Y^n = Z^n$  cannot have none zero integer solutions. This truth is known as the Fermat's Last Theorem.

In the equation  $X^n + Y^n = Z^n$ , when the numbers  $(X,Y,Z)$  are the natural numbers, the numbers  $(A=Z-Y)$  and  $(B=Z-X)$  are also the natural numbers.

### 2. General

**2-1.** What the equation  $X^n + Y^n = Z^n$  cannot have non zero integer solutions and what the equation  $X^n + Y = Z^n$  cannot have the natural number solutions are equivalent in meaning.

**[Explanation]**

When the number  $(n)$  is the even number,

$$(-U)^n + V^n = W^n$$

$$U^n + V^n = W^n.$$

When the number  $(n)$  is the odd number,

$$(-U)^n + V^n = W^n$$

$$-U^n+V^n=W^n$$

$$W^n+U^n=V^n.$$

**2-2.** When the equation  $X^n+Y^n=Z^n$  can have some natural number solutions, the natural numbers  $(X,Y,Z)$  need to be co-prime.

**[Explanation]**

$$U=QX, V=QY, W=QZ.$$

And

the natural numbers  $(X,Y,Z)$  are co-prime.

$$U^n+V^n=W^n$$

$$(QX)^n+(QY)^n=(QZ)^n$$

$$Q^n(X^n+Y^n=Z^n)$$

When

the number  $(Q)$  is the natural number,

the number  $(Q^n)$  is also the natural number.

### 3. Introduction

**3-1.** When  $A=Z-Y$  and  $B=Z-X$  in the equation  $X^n+Y^n=Z^n$ , we can find the equation

$$\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n.$$

**[Explanation]**

$$X^n+Y^n=Z^n$$

$$A=Z-Y, B=Z-X$$

$$Y+A=X+B=Z$$

$$X-A=Y-B=Z-A-B=X+Y-Z$$

And

$$(X-A)/(AB)^{1/n}=(Y-B)/(AB)^{1/n}=(Z-A-B)/(AB)^{1/n}=(X+Y-Z)/(AB)^{1/n}$$

This is G.

$$G=(X-A)/(AB)^{1/n}=(Y-B)/(AB)^{1/n}=(Z-A-B)/(AB)^{1/n}=(X+Y-Z)/(AB)^{1/n}$$

So,

$$X=G(AB)^{1/n}+A, Y=G(AB)^{1/n}+B, Z=G(AB)^{1/n}+A+B, X+Y-Z=G(AB)^{1/n}$$

Therefore,

$$\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$$

**3-2.** When  $n=1$ ,  $G=0$  and when  $n=2$ ,  $G=2^{1/2}>0$  and when  $n=3$ ,  $G=\text{Function}(A,B)>0$ .

**[Explanation]**

When  $n=1$ ,  $G=0$ .

$$(X+Y)=Z$$

$$X+Y-Z=G(AB)=0, G=0.$$

$$\{G(AB)+A\}+\{G(AB)+B\}=\{G(AB)+A+B\}$$

$$G(AB)=0$$

$$G=0$$

When  $n=2$ ,  $G=2^{1/2}>0$ .

$$(X^2+Y^2)>Z^2$$

$$X+Y-Z=G(AB)^{1/2}>0, G>0.$$

$$\{G(AB)^{1/2}+A\}^2+\{G(AB)^{1/2}+B\}^2=\{G(AB)^{1/2}+A+B\}^2$$

$$G^2(AB)=2AB$$

$$G=2^{1/2}>0$$

When  $n=3$ ,  $G=\text{Function}(A,B)>0$ .

$$(X^3+Y^3)>Z^3$$

$$X+Y-Z=G(AB)^{1/3}>0, G>0.$$

$$\{G(AB)^{1/3}+A\}^3+\{G(AB)^{1/3}+B\}^3=\{G(AB)^{1/3}+A+B\}^3$$

$$G=\text{Function}(A,B)>0$$

Generally,

we cannot find the solutions to the figure  $\{G(AB)^{1/3}\}$ .

## 4. 1st method about Fermat's Last Theorem proof

**4-1.** Transformation

$$X^n+Y^n=Z^n$$

$$(X^{n/2})^2+(Y^{n/2})^2=(Z^{n/2})^2$$

$$a=Z^{n/2}-Y^{n/2}, b=Z^{n/2}-X^{n/2}$$

$$\{G(ab)^{1/2}+a\}^2+\{G(ab)^{1/2}+b\}^2=\{G(ab)^{1/2}+a+b\}^2$$

$$G=2^{1/2}>0$$

$$X^{n/2}=(2ab)^{1/2}+a, Y^{n/2}=(2ab)^{1/2}+b, Z^{n/2}=(2ab)^{1/2}+a+b$$

**4-2.** When the number (n) is the even number in the equation  $X^n+Y^n=Z^n$ , the numbers (X,Y,Z) cannot be the natural numbers.

**4-2-1.** All Pythagorean triples.

**[Explanation]**

$$X^n+Y^n=Z^n$$

$$A=Z-Y, B=Z-X$$

$$X=(2AB)^{1/2}+A, Y=(2AB)^{1/2}+B, Z=(2AB)^{1/2}+A+B$$

When the numbers (X,Y,Z) are the natural numbers,  
the numbers (A,B) are also the natural numbers.

Therefore,

$$\text{when } 2AB=k^2(k \text{ is } 1,2,3\dots),$$

the natural numbers (X,Y,Z) are all Pythagorean triples.

**4-2-2.** All Pythagorean triples cannot be the power numbers.

**[Explanation]**

$$X=(2AB)^{1/2}+A, Y=(2AB)^{1/2}+B, Z=(2AB)^{1/2}+A+B$$

$$2AB=k^2(k \text{ is } 1,2,3\dots)$$

$$X=k+A=k(k+2B)/2B, Y=k+B=k(k+2A)/2A$$

And

$$XY=k(k+A)(k+2A)/2A=k(k+B)(k+2B)/2B$$

So,

the figure (XY) cannot be the power number in co-prime (A,B).

Therefore,

all Pythagorean triples cannot be the power numbers.

**4-2-3.** When the number (n) is the even number, the numbers  $(X^m, Y^m, Z^m)$  cannot be the natural numbers. So, the numbers (X,Y,Z) cannot be the natural numbers.

**[Explanation]**

$$X^n + Y^n = Z^n$$

$$(X^{n/2})^2 + (Y^{n/2})^2 = (Z^{n/2})^2$$

When  $n=2m$ ,

$$(X^m)^2 + (Y^m)^2 = (Z^m)^2.$$

The natural numbers  $(X^m, Y^m, Z^m)$  need to be the Pythagorean triples, but all Pythagorean triples cannot be the power numbers.

So,

the numbers  $(X^m, Y^m, Z^m)$  cannot be the natural numbers.

Therefore,

the numbers  $(X, Y, Z)$  cannot be the natural numbers.

**4-3.** When the number  $(n)$  is the odd number in the equation  $X^n + Y^n = Z^n$ , the numbers  $(X, Y, Z)$  cannot be the natural numbers.

**4-3-1.** The number  $(n)$  needs to be the odd and prime number.

**[Explanation]**

$$X^r + Y^r = Z^r$$

When

$r = nop..(n, o, p.. \text{ are the prime numbers.}),$

$$(X^{op..})^n + (Y^{op..})^n = (Z^{op..})^n.$$

When the numbers  $(X, Y, Z)$  are co-prime,

the numbers  $(X^{op..}, Y^{op..}, Z^{op..})$  are also co-prime.

Therefore,

when the numbers  $(X, Y, Z)$  are co-prime,

the number  $(n)$  needs to be the odd and prime number.

**4-3-2.** The figure  $(XY)^n$  is the natural number in the natural numbers  $(X, Y, Z)$ , but the figure  $[ab\{2a^2 + 2b^2 + 13ab + 6(a+b)(2ab)^{1/2}\}]$  cannot be the natural number in the natural numbers  $(X, Y, Z)$ . This is an apparent contradiction. Therefore, the numbers  $(X, Y, Z)$  cannot be the natural numbers, when the number  $(n)$  is the odd number.

[Explanation]

$$X^n + Y^n = Z^n$$

$$(X^{n/2})^2 + (Y^{n/2})^2 = (Z^{n/2})^2$$

$$a = Z^{n/2} - Y^{n/2}, \quad b = Z^{n/2} - X^{n/2}$$

$$X^{n/2} = (2ab)^{1/2} + a, \quad Y^{n/2} = (2ab)^{1/2} + b, \quad Z^{n/2} = (2ab)^{1/2} + a + b$$

$$(XY)^{n/2} = \{(2ab)^{1/2} + a\} \{(2ab)^{1/2} + b\}$$

Here,

the natural numbers  $(X, Y, Z)$  are co-prime

and the number  $(n)$  is the odd and prime number.

$$(XY)^n = ab \{2a^2 + 2b^2 + 13ab + 6(a+b)(2ab)^{1/2}\}$$

$$= (Z^{n/2} - X^{n/2})(Z^{n/2} - Y^{n/2}) \{2X^n + 2Y^n + 17Z^n - 17(ZX)^{n/2} - 17(ZY)^{n/2} + 13(XY)^{n/2} + 6(2^{1/2})(2Z^{n/2} - X^{n/2} - Y^{n/2})(Z^{n/2} - X^{n/2})^{1/2}(Z^{n/2} - Y^{n/2})^{1/2}\}$$

The figure  $(XY)^n$  is the natural number in the natural numbers  $(X, Y, Z)$ .

But

$$\text{the figure } [ab \{2a^2 + 2b^2 + 13ab + 6(a+b)(2ab)^{1/2}\}]$$

cannot be the natural number in the natural numbers  $(X, Y, Z)$ .

This is an apparent contradiction.

Therefore,

the numbers  $(X, Y, Z)$  cannot be the natural numbers.

**4-4.** Therefore, the equation  $X^n + Y^n = Z^n$  cannot have the natural number solutions in the even number  $(n)$  and in the odd number  $(n)$ .

## 5. 2nd method about Fermat's Last Theorem proof

### 5-1. Transformation

$$X^n + Y^n = Z^n$$

$$A = Z - Y, \quad B = Z - X$$

$$X + Y - Z = G(AB)^{1/n} > 0$$

$$\{G(AB)^{1/n} + A\}^n + \{G(AB)^{1/n} + B\}^n = \{G(AB)^{1/n} + A + B\}^n$$

**5-2.** In the equation  $\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$ , when the numbers (A,B) are the natural numbers, the figure  $\{G(AB)^{1/n}\}$  cannot be the natural number.

**5-2-1.** When the numbers (A,B) are the natural numbers, one each figure  $\{G(AB)^{1/n}\}$  is the positive number in the equation  $\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$ .

**[Explanation]**

$$\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$$

Generally,

we cannot solve this equation,

but

we see that we can have n each solutions

to the figure  $\{G(AB)^{1/n}\}$  in this equation.

When

the numbers (A,B) are the natural numbers,

one each figure  $\{G(AB)^{1/n}\}$  is the positive number

and

(n-1) each figures  $\{G(AB)^{1/n}\}$  cannot be the positive numbers.

**5-2-2.** The figure  $[G=\{2^{(n-2)/n}+\dots+2^{1/n}+1\}\{2A^{(n-2)}\}^{1/n}>0]$  cannot be the natural number in the equation  $\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$ , when the number (A) is the natural number and (A=B).

**[Explanation]**

$$\{G(AB)^{1/n}+A\}^n+\{G(AB)^{1/n}+B\}^n=\{G(AB)^{1/n}+A+B\}^n$$

When A=B,

$$2\{G+A^{(n-2)/n}\}^n=\{G+2A^{(n-2)/n}\}^n.$$

$$G=\{2^{(n-2)/n}+\dots+2^{1/n}+1\}\{2A^{(n-2)}\}^{1/n}=[\{2^{(n-2)/n}+\dots+2^{1/n}+1\}^n\{2A^{(n-2)}\}]^{1/n}$$

The figure  $[\{2^{(n-2)/n}+\dots+2^{1/n}+1\}^n]$  cannot be the natural number

and

the figure  $\{2A^{(n-2)}\}$  is the natural number in the natural number (A).

Therefore,

the formula  $[\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n}]$

cannot be the natural number in the natural number (A).

**5-2-3.** We make a numerical formula with the figure  $\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\}$ .

**[Explanation]**

$$\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

This numerical formula cannot be the natural number,

when the numbers (A,B) are the natural numbers.

Because

the figures  $\{2A^{(n-1)}B\}$  and  $\{2AB^{(n-1)}\}$  are the natural numbers,

when the numbers (A,B) are the natural numbers.

**5-2-4.** Here and now, we do the multiplication and the division to the formula  $\{G(AB)^{1/n}\}$  with the numerical formula that we made. So, we can find the formula (q) and the numerical formula  $\{G(AB)^{1/n}\}$ .

**[Explanation]**

$$q = 2G(AB)^{1/n} / \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

This is the numerical formula  $G(AB)^{1/n}$ .

$$G(AB)^{1/n} = q \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}] / 2$$

**5-2-5.** When  $A=B$ ,  $q=1$ .

**[Explanation]**

In the equation

$$\{G(AB)^{1/n} + A\}^n + \{G(AB)^{1/n} + B\}^n = \{G(AB)^{1/n} + A + B\}^n$$

When  $A=B$ ,

$$G = \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n}.$$

In the numerical formula  $\{G(AB)^{1/n}\}$ ,

$$G(AB)^{1/n} = q \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}] / 2.$$

When  $A=B$ ,

$$G = q \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n}.$$

So,

$$\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n} = q \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} \{2A^{(n-2)}\}^{1/n}.$$

Therefore,

when  $A=B$ ,  $q$  must be 1.

**5-2-6.** The figure  $\{G(AB)^{1/n}\}$  cannot be the natural number in the natural numbers  $(A,B)$ .

**[Explanation]**

$$q = 2G(AB)^{1/n} / \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}]$$

If

the figure  $\{G(AB)^{1/n}\}$  can be the natural number

in some natural numbers  $(A,B)$ ,

the formula  $\{2G(AB)^{1/n}\}$  cannot have the figure  $\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\}$ .

But

the numerical formula that we made

$$\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} [\{2A^{(n-1)}B\}^{1/n} + \{2AB^{(n-1)}\}^{1/n}],$$

have the figure  $\{2^{(n-2)/n} + \dots + 2^{1/n} + 1\}$ .

When  $A=B$ ,

$$q = 2G(AB)^{1/n} / \{2^{(n-2)/n} + \dots + 2^{1/n} + 1\} (2^{1/n} A).$$

So,

when  $A=B$ , the figure  $(q)$  cannot be 1.

This is an apparent contradiction.

Therefore,

the figure  $\{G(AB)^{1/n}\}$  cannot be the natural number

in the natural numbers  $(A,B)$ .

**5-3.** Therefore, the equation  $X^n + Y^n = Z^n$  cannot have the natural number solutions.

## 6. Conclusion

In the equation  $X^n + Y^n = Z^n$ , when  $n=2$ ,

$$X+Y-Z=(2AB)^{1/2}>0.$$

$$X=(2AB)^{1/2}+A, Y=(2AB)^{1/2}+B, Z=(2AB)^{1/2}+A+B$$

In the natural numbers  $(A=Z-Y)$ ,  $(B=Z-X)$

and  $\{2AB=k^2(k \text{ is } 1,2,3, \dots)\}$ ,

the natural numbers  $(X,Y,Z)$  are all Pythagorean triples.

In the equation  $X^n+Y^n=Z^n$ , when the number  $(n)$  is greater or equal 3,

$$X+Y-Z=G(AB)^{1/n}>0.$$

The equation  $X^n+Y^n=Z^n$  cannot have the natural number solutions.

It means that

the equation  $X^n+Y^n=Z^n$  cannot have non zero integer solutions.

## Acknowledgment

We believe in the Fermat.

And we believe that the space and the matters come into existence, when the numbers come into existence and we also believe that all cosmic materials and lives change but the number theory cannot change now and forever.

Thanks.

## References

**BarryCipra.**(1994)."Straightening out nonlinear codes." What's happening in Mathematical Sciences. pp. 37-40, vol. 2.

**Author1** 이재율 (Jae Yul Lee)

소속 Gyeonggy Electric Safety Company

주요관심분야 전기수학, 전력기술인, 주택관리사, 주식투자상담사

연락처 031-721-6190 (leejaeyul5@yahoo.co.kr)

**Author2** 이유진 (You Jin Lee)

소속 College of Veterinary Medicine Seoul National University

주요관심분야 개인수학교육, 자연환경, 주식투자

연락처 042-621-4848 (jgyoujin@hanmail.net)